

Primordial massive gravitational waves from Einstein-Chern-Simons-Weyl gravity

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Abstract. We investigate the evolution of cosmological perturbations during de Sitter inflation in the Einstein-Chern-Simons-Weyl gravity. Primordial massive gravitational waves are composed of one scalar, two vector and four tensor circularly polarized modes. We show that the vector power spectrum decays quickly like a transversely massive vector in the superhorizon limit $z \rightarrow 0$. In this limit, the power spectrum coming from massive tensor modes decays quickly, leading to the conventional tensor power spectrum. Also, we find that in the limit of $m^2 \rightarrow 0$ (keeping the Weyl-squared term only), the vector and tensor power spectra disappear. It implies that their power spectra are not gravitationally produced because they (vector and tensor) are decoupled from the expanding de Sitter background, as a result of conformal invariance.

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1 Introduction

The recent detection of primordial gravitational waves (GWs) via B-mode polarization of Cosmic Microwave Background Radiation (CMBR) by BICEP2 [1] has shown that the cosmic inflation at a high scale of 10^{16} GeV is the most plausible source of generating primordial GWs. The primordial GWs can be imprinted in the anisotropies and polarization spectrum of CMBR by making the photon redshifts. The B-mode signal observed by BICEP2 might also contain contributions from other sources (vector modes, cosmic strings) in addition to tensor modes [2].

The prediction about B-modes from inflation implies the phenomenon of GWs as well as quantum gravity. In order to explore this situation explicitly, we would like to mention that no conventional experiment is capable of detecting individual gravitons (quanta of the gravitational field) like photons (quanta of the electromagnetic field) because the LIGO is supposed to detect GWs (h_+ , h_\times) with a strain amplitude of 10^{-21} which amounts to 3×10^{37} gravitons [3]. This implies that if the LIGO wants to detect a single graviton, its sensitivity should be improved by a factor of the order of 3×10^{37} . The inflation implies a brief period during which the universe underwent an exponential expansion. If inflation occurred, however, the universe affords an access to detect gravitons because the inflation is considered as an ideal graviton amplifier to produce primordial GWs [4]. In this way, the inflation produces a classical signal of macroscopic GWs in response to spontaneous emission of gravitons. A classical signal of GWs may be considered as a coherent superposition of a large number of gravitons. This is similar to the LASER (light amplification by stimulated emission of radiation) which is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. In this sense, one may regard the inflation as a process of GWASG (gravitational wave amplification by spontaneous emission of gravitons). The difference is that the light (GW) is amplified by stimulated (spontaneous) emission of photons (gravitons). The simplest effect of primordial GWs is to produce a direct quadrupole anisotropy in the CMBR, inducing B-mode polarizations through Thomson

scattering. Furthermore, the mechanism of cosmic inflation naturally generates a stochastic background of primordial GWs which is an incoherent superposition of GWs [5].

The quadrupole anisotropy usually arises from 3 types of cosmological perturbations in Einstein gravity: scalar (due to density fluctuations); vector (due to vorticity induced by defects/strings); tensor (due to gravity waves). The curl-free E-mode may be due to both the scalar and tensor perturbations, whereas the B-mode is due to only vector or tensor perturbations because of their handedness.

A genuine massive gravity provides more physically propagating modes than the Einstein gravity: 5 and 2 tensor modes. If the graviton is massive, we expect that they will leave a different signature on the CMBR anisotropy spectrum. Similarly, it seems that there is no way to detect a single massive graviton directly by LIGO even though it has 5 degrees of freedom (DOF). However, there were some probes into a stochastic massive gravitational wave background which is an incoherent superposition of massive GWs produced by many unsolved astronomical source or by inflation [6, 7]. In this case, the observation of 6 polarization modes (+, ×, ○, ℓ, x, y) is an essential tool to probe for the massive gravity. Here + (plus) and × (cross) modes are tensor-type (spin-2) GWs, ○ (breathing) and ℓ (longitudinal) are scalar-type (spin-0) GWs. x and y are vector-type (spin-1) GWs.

In a massive gravity of Einstein-Chern-Simons-Weyl (ECSW) gravity, however, one has $7[1(\text{scalar})+2(\text{vector})+2(\text{tensor})+2(\text{massive tensor})]$ modes [8–10] because the Weyl-squared term could eliminate a longitudinal scalar and the equation of motion for tensor modes is fourth order.

If the massive graviton exists, its existence could be proved by inflation which may play the role of a massive graviton amplifier to obtain primordial massive GWs. That is, one method of computing massive GWs traces their origin to spontaneous emission of single massive gravitons, which got then amplified classically by inflation (expansion) into massive GWs imprinted in the CMBR temperature and polarization. When one compares massive GWs with GWs, the difference is twofold: the presence of graviton mass m and number of polarization modes. For the cosmological perturbation of a massive gravity, one introduces $SO(3)$ decomposition to a metric tensor which leads to six modes of two scalars, one vector with 2 modes, and a tensor with 2 under the newtonian gauge. Usually, the tensor perturbation produces both EE- and BB-mode polarization power spectra. The vector modes disappear in the inflationary background of Einstein gravity, while it can be propagating in the inflationary background of a massive gravity theory with Weyl term [11]. Two propagating vector modes reflect that the considering theory belongs to a massive gravity. The two scalar modes of $\Phi = -\Psi$ is combined with the inflaton $\delta\phi$ to give the comoving curvature perturbation $\mathcal{R} = -\Phi + (H/\dot{\phi})\delta\phi$ in the Einstein gravity, while they could become a propagating mode in the ECSW gravity. However, the Chern-Simons term does not contribute to the scalar sector because of the parity symmetry in scalar modes.

In this work, we propose the ECSW gravity theory as a massive gravity to detect massive GWs arisen from inflation. The cosmological perturbation of Einstein-Weyl gravity has been performed by showing mainly that the vector perturbation cannot be neglected [11]. This is easily understood when one recognizes that the Einstein-Weyl gravity describes a massive gravity with 7 DOF. We will focus on computing all power spectra by performing the cosmological perturbations around the de Sitter inflation. This theory possesses a tensor ghost as massive GWs when one compares with the dRGT massive gravity [12, 13].

2 ECSW gravity

Let us first consider the Einstein-Chern-Simons-Weyl (ECSW) gravity whose action is given by

$$S_{\text{ECSW}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[R - 2\kappa\Lambda - \kappa(\partial\phi)^2 - \kappa m_\phi^2 \phi^2 + \frac{1}{4} \theta {}^*RR - \frac{1}{2m^2} C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} \right], \quad (2.1)$$

where the Chern-Simons term and Weyl-squared term take the form as

$${}^*RR = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} R^{\mu\nu}_{\alpha\beta} R_{\gamma\delta\mu\nu}, \quad (2.2)$$

$$C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} = 2 \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) + (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2). \quad (2.3)$$

For our purpose, we include a massive scalar ϕ as a competitor. Here we have $\kappa = 8\pi G = 1/M_{\text{P}}^2$, M_{P} being the reduced Planck mass. Greek indices run from 0 to 3 with conventions $(-+++)$, while Latin indices run from 1 to 3. We note that the Chern-Simons term is coupled to not a scalar ϕ but a Chern-Simons scalar θ , implying the non-dynamical Chern-Simons gravity theory [14, 15]. This contrasts to a conventional cosmological approach obtained from the dynamical Chern-Simons gravity with $f(\phi) {}^*RR$ [16–19]. Further, we note that the Weyl-squared term is invariant under the conformal transformation of $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$ like the Maxwell kinetic term of $-F^2/4$ which implies that the vector and tensor perturbations are decoupled from the de Sitter inflation in the limit of $m^2 \rightarrow 0$ (keeping the Weyl-squared term only).

The Einstein equation is given by

$$G_{\mu\nu} + \kappa\Lambda g_{\mu\nu} + \mathcal{C}_{\mu\nu} - \frac{1}{m^2} B_{\mu\nu} = \kappa T_{\mu\nu} \quad (2.4)$$

where the Einstein tensor $G_{\mu\nu}$, the Cotton tensor $\mathcal{C}_{\mu\nu}$, the Bach tensor $B_{\mu\nu}$, and the energy-momentum tensor $T_{\mu\nu}$ take the forms

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \quad (2.5)$$

$$\mathcal{C}_{\mu\nu} = \nabla_\gamma \theta \epsilon^{\gamma\rho\sigma}{}_{(\mu} \nabla_{|\sigma|} R_{\nu)\rho} + \frac{1}{2} \nabla_\gamma \nabla_\rho \theta \epsilon_{(\nu}{}^{\gamma\sigma\delta} R^{\rho}{}_{\mu)\sigma\delta}, \quad (2.6)$$

$$B_{\mu\nu} = 2\nabla^\rho \nabla^\sigma C_{\mu\rho\nu\sigma} + G^{\rho\sigma} C_{\mu\rho\nu\sigma}, \quad (2.7)$$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\rho \phi \partial^\rho \phi + \frac{m_\phi^2}{2} \phi^2 \right). \quad (2.8)$$

On the other hand, the scalar equation leads to

$$\nabla^2 \phi - m_\phi^2 \phi = 0, \quad (2.9)$$

while the divergence of the left-hand side of (2.4) should be zero by imposing the Bianchi identity as

$$\nabla^\mu \mathcal{C}_{\mu\nu} = -\frac{\nabla_\nu \theta}{8} {}^*RR = 0 \quad (2.10)$$

because $\nabla^\mu T_{\mu\nu} = 0$ implies (2.9) and $\nabla^\mu B_{\mu\nu} = 0$.

For a conformally flat Friedmann-Robertson-Walker (FRW) background expressed by the conformal time η

$$ds_{\text{FRW}}^2 = a(\eta)^2 \left[-d\eta^2 + \delta_{ij} dx^i dx^j \right], \quad (2.11)$$

the Einstein equation and scalar equation are given by

$$\mathcal{H}^2 = \frac{\kappa}{3} \left(a^2 \Lambda + \frac{1}{2} (\phi')^2 + \frac{a^2}{2} m_\phi^2 \phi^2 \right), \quad (2.12)$$

$$\phi'' + 2\mathcal{H}\phi' + a^2 m_\phi^2 \phi = 0, \quad (2.13)$$

where ' (prime) denotes differentiation with respect to conformal time η and $\mathcal{H} = a'/a$. Here we note that there are no contributions from the Cotton and Bach tensors because the Cotton tensor represents a parity-violating term and Bach tensor comes from a conformal invariant Weyl-squared term, while the FRW universe preserves the parity symmetry. Also, Eq.(2.10) is satisfied, because the Pontryagin constraint ($*RR = 0$) is preserved on the FRW background for $\nabla_\nu \theta \neq 0$. This constraint is also recovered when one varies the action (2.1) by the field θ . It implies that a non-dynamical field θ remains unfixed in the background evolution, but it could be determined when one solves the tensor perturbed equation (see Sec. 3.3)

Now one starts with general perturbed metric

$$ds^2 = a(\eta)^2 \left[- (1 + 2\Psi) d\eta^2 + 2B_i d\eta dx^i + (\delta_{ij} + \bar{h}_{ij}) dx^i dx^j \right], \quad (2.14)$$

where the SO(3)-decomposition is given by

$$B_i = \partial_i B + \Psi_i, \quad \bar{h}_{ij} = 2\Phi \delta_{ij} + 2\partial_i \partial_j E + \partial_i \bar{E}_j + \partial_j \bar{E}_i + h_{ij} \quad (2.15)$$

with the transverse vectors $\partial_i \Psi^i = 0$, $\partial_i \bar{E}^i = 0$, and transverse-traceless tensor $\partial_i h^{ij} = h = 0$. To have 7 propagating modes implied by the massive gravity of ECSW (2.1), we first choose the Newtonian gauge of $B = E = 0$ and $\bar{E}_i = 0$ which leads to $12[10+2(\text{massive tensor})]-4=8$ modes. In this case, the corresponding perturbed metric and scalar can be written as

$$ds^2 = a(\eta)^2 \left[- (1 + 2\Psi) d\eta^2 + 2\Psi_i d\eta dx^i + \left\{ (1 + 2\Phi) \delta_{ij} + h_{ij} \right\} dx^i dx^j \right], \quad (2.16)$$

$$\phi = \bar{\phi} + \delta\phi. \quad (2.17)$$

Here $a(\eta)$ and $\bar{\phi} = 0$ denote the background spacetime of de Sitter inflation. In the case of Einstein gravity, one has a connection

$$\Psi = -\Phi \quad (2.18)$$

from the linearized Einstein equation of $\delta G_i{}^j = \partial_i \partial^j (\Psi + \Phi) = 0$ and they are not physical DOF. However, since the ECSW gravity is considered as a massive gravity with 7 DOF, we impose one constraint to meet the massive gravity with 7 DOF.

There are two ways to obtain the cosmological perturbed equations: one is to linearize the Einstein and scalar equation around the de Sitter inflation background directly and the other is first to obtain the bilinear action and then, varying it to obtain the perturbed equations. In this work, we choose the second approach.

Now we expand the ECSW action (2.1) up to quadratic order in the perturbations $(\Psi, \Phi, \delta\phi, \Psi_i, h_{ij})$ on the de Sitter background [11] then the bilinear actions for scalar, vector and tensor perturbations can be found as

$$\kappa S_{\text{ECSW}}^{(\text{S})} = \frac{1}{2} \int d^4x \left\{ a^2 \left[-6\Phi'^2 + 12\mathcal{H}\Psi\Phi' + 2\partial_i\Phi\partial^i\Phi + 4\partial_i\Phi\partial^i\Psi - 6\mathcal{H}^2\Psi^2 \right. \right. \\ \left. \left. + \kappa \left(\delta\phi'^2 - \partial_i\delta\phi\partial^i\delta\phi - a^2m_\phi^2\delta\phi^2 \right) \right] - \frac{2}{3m^2} [(\partial^2(\Psi - \Phi))^2] \right\}, \quad (2.19)$$

$$\kappa S_{\text{ECSW}}^{(\text{V})} = \frac{1}{4} \int d^4x \left[a^2 \partial_i\Psi_j\partial^i\Psi^j - \theta'\epsilon_i{}^{jk}\partial^\ell\Psi^i\partial_j\partial_\ell\Psi_k - \frac{1}{m^2}(\partial_i\Psi'_j\partial^i\Psi'^j - \partial^2\Psi_i\partial^2\Psi^i) \right], \quad (2.20)$$

$$\kappa S_{\text{ECSW}}^{(\text{T})} = \frac{1}{8} \int d^4x \left[a^2(h'_{ij}h'^{ij} - \partial_k h_{ij}\partial^k h^{ij}) - \theta'\epsilon_i{}^{jk}(h'^{\ell i}\partial_j h'_{k\ell} - \partial^\ell h^{pi}\partial_j h_{kp}) \right. \\ \left. - \frac{1}{m^2}(h''_{ij}h''^{ij} - 2\partial_k h'_{ij}\partial^k h'^{ij} + \partial^2 h_{ij}\partial^2 h^{ij}) \right], \quad (2.21)$$

where $\partial^2 \equiv \partial_i\partial^i$ and $\epsilon^{ijk} \equiv \epsilon^{0ijk}$.

Varying the actions (2.20) and (2.21) with respect to Ψ^i and h^{ij} , respectively leads to the equations of motion for vector and tensor perturbations as follows:

$$\square\Psi_i - m^2 a^2 \Psi_i - m^2 \theta' \epsilon_i{}^{j\ell} \partial_\ell \Psi_j = 0, \quad (2.22)$$

$$\square^2 h_{ij} - m^2 a^2 \square h_{ij} + 2m^2 a^2 \mathcal{H} h'_{ij} \\ - m^2 \epsilon_j{}^{\ell k} \left(\theta'' \partial_\ell h'_{ki} + \theta' \partial_\ell h''_{ki} - \theta' \partial_\ell \square h_{ki} \right) = 0. \quad (2.23)$$

We emphasize that Eqs.(2.22) and (2.23) are newly derived equations. Turning off the Weyl-squared term (in the limit of $m^2 \rightarrow \infty$), Eq.(2.22) is trivial which implies the non-propagating vector modes in the modified Chern-Simons gravity [16]. In the other limit of $m^2 \rightarrow 0$, we keep the Weyl-squared term only which is surely independent of a^2 because it is invariant under the conformal transformation of $g_{\mu\nu} \rightarrow a^2 \eta_{\mu\nu}$. This implies that the perturbed field equations are $\square\Psi_i = 0$ and $\square^2 h_{ij} = 0$ which are independent of the expanding background in the Weyl gravity.

Before we proceed, we briefly mention the scalar perturbations. The Chern-Simons term does not contribute to the scalar bilinear action (2.19) because of the parity symmetry in scalar modes. Also, it is important to note that when one compares the last terms in (2.19), (2.20), and (2.21), the last term in (2.19) is a purely (space-like) fourth-order term without the kinetic term. Hence it could be played the role of a constraint to reduce 2 DOF to one DOF. In this case, an elegant constraint is to choose

$$\Psi = \Phi \quad (2.24)$$

which corresponds to taking $m^2 \rightarrow \infty$ effectively. We note that the constraint (2.24) is different from $\Psi = -\Phi$ (2.18) obtained from the Einstein gravity. Requiring the condition of $\Psi = \Phi$, the bilinear scalar action (2.19) takes a simple form

$$\kappa \tilde{S}_{\text{ECSW}}^{(\text{S})} = \frac{1}{2} \int d^4x a^2 \left[-6\Phi'^2 + 12\mathcal{H}\Phi\Phi' + 6\partial_i\Phi\partial^i\Phi - 6\mathcal{H}^2\Phi^2 \right. \\ \left. + \kappa \left(\delta\phi'^2 - \partial_i\delta\phi\partial^i\delta\phi - a^2m_\phi^2\delta\phi^2 \right) \right] \quad (2.25)$$

which is our bilinear scalar action.

3 Perturbations on de Sitter inflation

In the de Sitter inflation with constant H and $\bar{\phi} = 0$, one has the background spacetime and Friedmann equation (2.12)

$$ds_{\text{dS}}^2 = -dt^2 + e^{2Ht} \delta_{ij} dx^i dx^j, \quad H^2 = \frac{\kappa \Lambda}{3} \equiv \frac{m_{\text{In}}^4}{3M_{\text{P}}^2} \quad (3.1)$$

which implies that

$$a(t) = e^{Ht} \rightarrow a(\eta) = -\frac{1}{H\eta}. \quad (3.2)$$

During the De Sitter stage, a goes from a very small to a very large value which corresponds to $\eta = -\frac{1}{aH}$ running from $-\infty$ to zero. Also, one has

$$\mathcal{H}^2 = \mathcal{H}' = a^2 H^2. \quad (3.3)$$

Even though the de Sitter inflation does not provide a graceful exit when one compares with the slow-roll inflation, we choose it for a simple computation.

The BICEP2 measurement of $r = A_T(k_*)/A_s(k_*) = 0.2$ together with PLANCK measurement of scalar amplitude $A_s = 2.215 \times 10^{-9}$ determines the scale m_{I} of inflation as [4, 20]

$$A_T(k_*) = \frac{2}{\pi^2} \left(\frac{H^2}{M_{\text{P}}^2} \right) \rightarrow H \simeq 1.1 \times 10^{14} \text{ GeV} \rightarrow V^{1/4} = m_{\text{I}} = 2.1 \times 10^{16} \text{ GeV} \quad (3.4)$$

with $M_{\text{P}} = 2.4 \times 10^{18} \text{ GeV}$. This is very close to the GUT scale. This implies the small bound

$$\frac{H^2}{M_{\text{P}}^2} = 2.1 \times 10^{-9} \ll 1. \quad (3.5)$$

3.1 Scalar perturbations

In order to investigate the scalar perturbation in the de Sitter background, we first derive the linearized scalar equations. Varying (2.25) with respect to Φ leads to

$$\Phi'' + 2\mathcal{H}\Phi' - 4\mathcal{H}^2\phi - \partial^2\Phi = 0. \quad (3.6)$$

Similarly, varying (2.25) with respect to $\delta\phi$, one has a massive equation

$$\delta\phi'' + 2\mathcal{H}\delta\phi' + m_\phi^2 a^2 \delta\phi - \partial^2\delta\phi = 0. \quad (3.7)$$

Considering the Fourier expansion of Φ and $\delta\phi$

$$(\Phi(\eta, \mathbf{x}), \delta\phi(\eta, \mathbf{x})) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} (\Phi_{\mathbf{k}}(\eta), \phi_{\mathbf{k}}(\eta)) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (3.8)$$

the equation (3.6) can be written as

$$\left[\frac{d^2}{d\eta^2} - \frac{2}{\eta} \frac{d}{d\eta} + k^2 - \frac{4}{\eta^2} \right] \Phi_{\mathbf{k}}(\eta) = 0. \quad (3.9)$$

Now we introduce $z = -\eta k$ and a new variable $v_{\mathbf{k}} = a\Phi_{\mathbf{k}}(\eta) \rightarrow \frac{k}{H} \frac{1}{z} \Phi_{\mathbf{k}}(z)$, then Eq.(3.9) takes a simple form as

$$\left[\frac{d^2}{dz^2} + 1 - \frac{6}{z^2} \right] v_{\mathbf{k}} = 0. \quad (3.10)$$

Considering $v_{\mathbf{k}} = \sqrt{z} \tilde{v}_{\mathbf{k}}$, Eq.(3.10) reduces to the Bessel's equation

$$\left[\frac{d^2}{dz^2} + \frac{1}{z} \frac{d}{dz} + 1 - \frac{\nu_{\Phi}^2}{z^2} \right] \tilde{v}_{\mathbf{k}}(z) = 0 \quad (3.11)$$

with

$$\nu_{\Phi} = \frac{5}{2}. \quad (3.12)$$

The solution is given by the Hankel function as

$$\Phi_{\mathbf{k}}(z) = \frac{\sqrt{z}}{a} H_{5/2}^{(1)}. \quad (3.13)$$

On the other hand, the scalar equation (3.7) is given by

$$\left[\frac{d^2}{d\eta^2} - \frac{2}{\eta} \frac{d}{d\eta} + k^2 + \frac{m_{\phi}^2}{H^2} \frac{1}{\eta^2} \right] \phi_{\mathbf{k}}(\eta) = 0, \quad (3.14)$$

which can be further transformed into

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{2}{\eta^2} + \frac{m_{\phi}^2}{H^2} \frac{1}{\eta^2} \right] \tilde{\phi}_{\mathbf{k}}(\eta) = 0 \quad (3.15)$$

for $\tilde{\phi}_{\mathbf{k}} = a\phi_{\mathbf{k}} = -\phi_{\mathbf{k}}/(H\eta)$. After expressing (3.15) in terms of $z = -k\eta$ and then introducing $\tilde{\phi}_{\mathbf{k}} = \sqrt{z} \tilde{\tilde{\phi}}_{\mathbf{k}}$, it leads to the Bessel's equation as

$$\left[\frac{d^2}{dz^2} + \frac{1}{z} \frac{d}{dz} + 1 - \frac{\nu_{\phi}^2}{z^2} \right] \tilde{\tilde{\phi}}_{\mathbf{k}}(z) = 0 \quad (3.16)$$

with

$$\nu_{\phi} = \sqrt{\frac{9}{4} - \frac{m_{\phi}^2}{H^2}}. \quad (3.17)$$

The solution to (3.16) is given by the Hankel function $H_{\nu}^{(1)}$. Accordingly, one has the solution to (3.14)

$$\phi_{\mathbf{k}}(z) = \frac{\sqrt{z}}{a} \tilde{\tilde{\phi}}_{\mathbf{k}} = \frac{\sqrt{z}}{a} H_{\nu_{\phi}}^{(1)}(z). \quad (3.18)$$

3.2 Vector perturbations

We first consider Eq.(2.22) for vector perturbation and expand the mode Ψ_i in plane waves with the right-handed and left-handed circularly polarized states

$$\Psi_i(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \sum_{s=R,L} \tilde{p}_i^s(\mathbf{k}) \Psi_{\mathbf{k}}^s(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (3.19)$$

where \tilde{p}_i^s are the polarization vectors for the right-handed or left-handed circularly polarized state. They are defined by $\tilde{p}_i^{R/L} = \frac{1}{\sqrt{2}}(p_i^1 \pm ip_i^2)$ with $p_i^{1/2}$ linear polarization vectors. We note that $\tilde{p}_i^{R/L}(\tilde{p}^{R/L,i})^* = 1$, while $p_i^{1/2}p^{1/2,i} = 1$. Also, circularly polarized vector mode $\Psi_{\mathbf{k}}^s$ are defined by $\Psi_{\mathbf{k}}^s = \frac{1}{\sqrt{2}}(v_{\mathbf{k}}^1 \mp iv_{\mathbf{k}}^2)$ with $v_{\mathbf{k}}^{1/2}$ linearly polarized vector modes.

Plugging (3.19) into the equation (2.22), one finds the equation

$$\left[\frac{d^2}{d\eta^2} + k^2 + m^2 \left(\frac{1}{\eta^2 H^2} - \lambda^s k \frac{d\theta}{d\eta} \right) \right] \Psi_{\mathbf{k}}^s(\eta) = 0, \quad (3.20)$$

where $\lambda^{R/L} = \pm 1$. In deriving Eq.(3.20), the following relation was used:

$$ik_c \epsilon_a^{cd} \tilde{p}_d^s = \lambda^s k \tilde{p}_a^s. \quad (3.21)$$

At this stage, we choose a non-dynamical field θ to solve (3.20). We mention that θ remains unfixed in the background evolution, but it must be determined when one tries to solve the vector perturbed equation (3.20). Note that in the inflation model with the Gauss-Bonnet and the parity violating corrections, it is given by $\theta = c \ln \eta$ [19] to have slow-roll inflation, while it will take the form $\theta = c/\eta$ in the ECSW gravity to make factorization of fourth-order tensor equation (see Sec.3.3). Now we choose $\theta = c \ln \eta$, then one has $\theta' = c/\eta$. In this case, Eq.(3.20) takes the form

$$\left[\frac{d^2}{d\eta^2} + k^2 + m^2 \left(\frac{1}{\eta^2 H^2} - \frac{\lambda^s k c}{\eta} \right) \right] \Psi_{\mathbf{k}}^s(\eta) = 0, \quad (3.22)$$

which could describe a propagation of circularly polarized vector waves.

For $\theta = c/\eta$, however, one has $\theta' = -c/\eta^2$. In this case, Eq.(3.20) reduces to

$$\left[\frac{d^2}{d\eta^2} + k^2 + m^2 \left(\frac{1}{\eta^2 H^2} + \frac{\lambda^s k c}{\eta^2} \right) \right] \Psi_{\mathbf{k}}^s(\eta) = 0, \quad (3.23)$$

which is the same as the massive tensor equation (3.38). The above shows that the vector equation depends on the choice of θ .

Finally, for $\theta = 0$, Eq.(3.20) reduces to

$$\left[\frac{d^2}{d\eta^2} + k^2 + \frac{m^2}{\eta^2 H^2} \right] \Psi_{\mathbf{k}}(\eta) = 0, \quad (3.24)$$

which is just the massive equation for the transverse vector \mathcal{A}^\perp [21], while the equation of longitudinal vector \mathcal{A}^\parallel leads to the scalar equation (3.15) when we consider the massive Maxwell Lagrangian after plugging $g_{\mu\nu} = a^2 \eta_{\mu\nu}$

$$\mathcal{L}_M = \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{m_F^2}{2} a^2 A^\mu A_\mu \right). \quad (3.25)$$

Here the first term preserves conformal symmetry like the Weyl-squared term, while the second term breaks the conformal symmetry. In the limit of $m_F^2 \rightarrow 0$, one recovers the conformally invariant Maxwell term.

3.3 Tensor perturbations

Now we turn to the equation (2.23) for tensor perturbations. In this case, the metric tensor h_{ij} can be expanded in Fourier modes

$$h_{ij}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \sum_{s=R,L} \tilde{p}_{ij}^s(\mathbf{k}) h_{\mathbf{k}}^s(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (3.26)$$

where \tilde{p}_{ij}^s are the polarization tensors for the right-handed or left-handed circularly polarized state. They are defined by $\tilde{p}_{ij}^{R/L} = \frac{1}{\sqrt{2}}(p_{ij}^+ \pm ip_{ij}^\times)$ with $p_{ij}^{+/\times}$ linear polarization tensors. We note that $\tilde{p}_{ij}^{R/L}(\tilde{p}^{R/L,ij})^* = 1$, while $p_{ij}^{+/\times} p^{+/\times,ij} = 1$. Also, circularly polarized tensor mode $h_{\mathbf{k}}^s$ is defined by $h_{\mathbf{k}}^s = \frac{1}{\sqrt{2}}(h_{\mathbf{k}}^+ \mp ih_{\mathbf{k}}^\times)$ with $h_{\mathbf{k}}^{+/\times}$ linearly polarized tensor modes.

Plugging (3.26) into (2.23) leads to the fourth-order differential equation

$$\begin{aligned} (h_{\mathbf{k}}^s)'''' + 2k^2(h_{\mathbf{k}}^s)'' + k^4 h_{\mathbf{k}}^s + m^2(a^2 - \lambda^s k \theta')(h_{\mathbf{k}}^s)'' \\ + m^2(2a^2 \mathcal{H} - \lambda^s k \theta'')(h_{\mathbf{k}}^s)' + m^2(a^2 - \lambda^s k \theta')k^2 h_{\mathbf{k}}^s = 0, \end{aligned} \quad (3.27)$$

where we used

$$ik_c \epsilon_a^{cd} \tilde{p}_{bd}^s = \lambda^s k \tilde{p}_{ab}^s. \quad (3.28)$$

It is important to note that factorizing the fourth-order equation (3.27) into two second-order equations is a nontrivial task because the Chern-Simons field θ and its derivatives are present.

In the limit of $m^2 \rightarrow \infty$, one recovers the tensor perturbation equation for the Chern-Simons modified gravity which is surely a second-order equation [17]

$$\left(1 - \lambda^s k \frac{\theta'}{a^2}\right)(h_{\mathbf{k}}^{s,CS})'' + \left(2\mathcal{H} - \lambda^s k \frac{\theta''}{a^2}\right)(h_{\mathbf{k}}^{s,CS})' + \left(1 - \lambda^s k \frac{\theta'}{a^2}\right)k^2 h_{\mathbf{k}}^{s,CS} = 0, \quad (3.29)$$

which is transformed into the Mukhanov-Sasaki type equation

$$(\mu_{\mathbf{k}}^{s,CS})'' + \left(k^2 - \frac{z_s''}{z_s}\right)\mu_{\mathbf{k}}^{s,CS} = 0 \quad (3.30)$$

when one introduces

$$z_s(\eta, k) = a\sqrt{1 - \lambda^s k \frac{\theta'}{a^2}}, \quad \mu_{\mathbf{k}}^{s,CS} = z_s h_{\mathbf{k}}^{s,CS}. \quad (3.31)$$

The effective potential z''/z depends not only on time η and polarization λ^s , but also on the wave number k , which shows a difference when comparing with the standard case $z(\eta)$ [18]. For $\theta = c \ln \eta$, one has $\theta'/a^2 = cH^2\eta$, where $c = -\Omega/(M_c H)$ [19]. In this case, Eq.(3.30) takes the form

$$(\mu_{\mathbf{k}}^{s,CS})'' + \left(k^2 - \frac{2}{\eta^2} + \frac{\lambda_s k H \Omega}{M_c} \frac{1}{\eta}\right)\mu_{\mathbf{k}}^{s,CS}(\eta) = 0, \quad (3.32)$$

which could describe a propagation of circularly polarized GWs. Here $\Omega = M_c \dot{\theta}/(2M_{\text{Pl}}^2)$ was considered to be a nearly constant with $M_c = k/a$.

For $\theta = c/\eta$, however, one has $\theta'/a^2 = -cH^2$ which implies $z_s = a\sqrt{1 + \lambda_s k c H^2}$. Then, Eq.(3.30) reduces to

$$(\mu_{\mathbf{k}}^{s,CS})'' + \left(k^2 - \frac{2}{\eta^2}\right)\mu_{\mathbf{k}}^{s,CS}(\eta) = 0 \quad (3.33)$$

which is just the tensor perturbation equation (3.37). The above shows that the tensor perturbed equation depends on the choice of θ .

However, it is shown that the Eq.(3.27) can be factorized as the following two different types (see Appendix):

$$\left[\frac{d^2}{d\eta^2} + \frac{2}{\eta} \frac{d}{d\eta} + k^2 + m^2 \left(\frac{1}{\eta^2 H^2} - \lambda^s k \frac{d\theta}{d\eta} \right) \right] \left[\frac{d^2}{d\eta^2} - \frac{2}{\eta} \frac{d}{d\eta} + k^2 \right] h_{\mathbf{k}}^s = 0, \quad (3.34)$$

$$\left[\frac{d^2}{d\eta^2} - \frac{2}{\eta} \frac{d}{d\eta} + k^2 \right] \left[\eta^2 \frac{d^2}{d\eta^2} - 2\eta \frac{d}{d\eta} + 2 + k^2 \eta^2 + m^2 \left(\frac{1}{H^2} - \lambda^s k \eta^2 \frac{d\theta}{d\eta} \right) \right] h_{\mathbf{k}}^s = 0 \quad (3.35)$$

when one chooses

$$\theta = c_1 + \frac{c_2}{\eta} \quad (3.36)$$

with integration constants c_1 and c_2 . Their mass dimensions of c_1 and c_2 are given by $[M]^{-2}$ and $[M]^{-3}$, respectively. The choice of (3.36) contrasts to the dynamical Chern-Simons coupling studied in [16–19].

Introducing a new quantity $\mu_{\mathbf{k}}^s$ defined by $h_{\mathbf{k}}^s = \eta \mu_{\mathbf{k}}^s$, one can read off the Einstein (E) and Chern-Simons-Weyl (CSW) mode equations from Eqs.(3.34) and (3.35) as

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{2}{\eta^2} \right] \mu_{\mathbf{k}}^{s(E)} = 0, \quad (3.37)$$

$$\left[\frac{d^2}{d\eta^2} + k^2 + m^2 \left(\frac{1}{\eta^2 H^2} - \lambda^s k \frac{d\theta}{d\eta} \right) \right] \mu_{\mathbf{k}}^{s(\text{CSW})} = 0. \quad (3.38)$$

We note that Eq.(3.38) is exactly the same form as Eq.(3.20), obtained for the vector perturbation $\Psi_{\mathbf{k}}^s$ where θ is undetermined. For $c_1 = 0$ and $c_2 = M_{\text{P}}^{-3}$ together with $z = -k\eta$ [17], Eqs. (3.37) and (3.38) take the forms

$$\left[\frac{d^2}{dz^2} + 1 - \frac{2}{z^2} \right] \mu_{\mathbf{k}}^{s(E)} = 0, \quad (3.39)$$

$$\left[\frac{d^2}{dz^2} + 1 + \frac{\tilde{m}_s^2}{H^2} \frac{1}{z^2} \right] \mu_{\mathbf{k}}^{s(\text{CSW})} = 0 \quad (3.40)$$

with

$$\tilde{m}_s^2 = m^2 (1 + \lambda^s k H^2 M_{\text{P}}^{-3}). \quad (3.41)$$

It is easy to show that the tensor solution to (3.39) is given by

$$\mu_{\mathbf{k}}^{s(E)} = \alpha e^{iz} \left(1 + \frac{i}{z} \right) + \beta e^{-iz} \left(1 - \frac{i}{z} \right), \quad (3.42)$$

where α and β are the undetermined normalization constants.

Introducing

$$\mu_{\mathbf{k}}^{s(\text{CSW})} = \sqrt{z} \tilde{\mu}_{\mathbf{k}}^{s(\text{CSW})}, \quad (3.43)$$

Eq.(3.40) reduces to the Bessel equation

$$\left[\frac{d^2}{dz^2} + \frac{1}{z} \frac{d}{dz} + 1 - \frac{\nu_s^2}{z^2} \right] \tilde{\mu}_{\mathbf{k}}^{s(\text{CSW})} = 0 \quad (3.44)$$

whose solution is given by the Hankel function

$$\tilde{\mu}_{\mathbf{k}}^{s(\text{CSW})} = H_{\nu_s}^{(1)}(z), \quad \nu_s = \sqrt{\frac{1}{4} - \frac{\tilde{m}_s^2}{H^2}} < \frac{1}{2}. \quad (3.45)$$

Considering the bound (3.5), we expect to have $kH^2/M_{\text{p}}^3 \ll 1$. It means that we can treat the parity-violating effect as a small correction in Eq. (3.40). Here, requiring that the index ν_s be positive leads to the condition

$$\frac{\tilde{m}_s^2}{H^2} < \frac{1}{4} \rightarrow m^2 < \frac{H^2}{4} \rightarrow m < 5.5 \times 10^{13} \text{GeV} \quad (3.46)$$

which corresponds to the graviton mass bound.

As a byproduct, if the Einstein-mode equation (3.39) is expressed in terms of the Bessel equation, it gives $\nu_s = 3/2$. Its solution is found to be

$$\mu_{\mathbf{k}}^{s(\text{E})} = \sqrt{z} H_{3/2}^{(1)}(z) = \sqrt{\frac{2}{\pi}} e^{-i\pi} e^{iz} \left(1 + \frac{i}{z}\right) \quad (3.47)$$

which is the first term of (3.42), while the second term is given by $\sqrt{z} H_{3/2}^{(2)}(z)$.

Finally, the two tensor modes are given by

$$\left\{ h_{\mathbf{k}}^{s(\text{E})}(z), h_{\mathbf{k}}^{s(\text{CSW})}(z) \right\} = \left\{ z^{\frac{3}{2}} H_{3/2}^{(1)}(z), z^{\frac{3}{2}} H_{\nu_s}^{(1)}(z) \right\}. \quad (3.48)$$

For later convenience, we list asymptotic forms of the Hankel function

$$H_{\nu_s}^{(1)}(z) \Big|_{z \rightarrow \infty} \sim \sqrt{\frac{2}{\pi z}} e^{i(z - \frac{\nu_s \pi}{2} - \frac{\pi}{4})}, \quad H_{\nu_s}^{(1)}(z) \Big|_{z \rightarrow 0} \sim \frac{i \Gamma(\nu_s)}{\pi} \frac{1}{(\frac{z}{2})^{\nu_s}}. \quad (3.49)$$

4 Primordial power spectra

The power spectrum is usually given by the two-point correlation function which is calculated in the vacuum state $|0\rangle$. It is defined by

$$\langle 0 | \mathcal{F}(\eta, \mathbf{x}) \mathcal{F}(\eta, \mathbf{x}') | 0 \rangle = \int d^3 \mathbf{k} \frac{\mathcal{P}_{\mathcal{F}}}{4\pi k^3} e^{-i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}, \quad (4.1)$$

where \mathcal{F} denotes a scalar, vector, and tensor and $k = |\mathbf{k}|$ is the wave number. Fluctuations are created on all length scales with wave number k . Cosmologically relevant fluctuations start their lives inside the Hubble radius which defines the subhorizon as

$$\text{subhorizon : } k \gg aH \quad (z = -k\eta \gg 1). \quad (4.2)$$

On the other hand, the comoving Hubble radius $(aH)^{-1}$ shrinks during inflation while the comoving wavenumber k is constant. Therefore, eventually all fluctuations exit the comoving Hubble radius which defines the superhorizon as

$$\text{superhorizon : } k \ll aH \quad (z = -k\eta \ll 1). \quad (4.3)$$

We might calculate the quantum-mechanical variance of fluctuations (two-point function) by taking the Bunch-Davies vacuum $|0\rangle$ in the de Sitter inflation. In the de Sitter inflation, we choose the limit of $z \rightarrow \infty$ (subhorizon) to define the Bunch-Davies vacuum, while we choose the limit of $z \rightarrow 0$ to obtain a definite form of power spectra.

In general, all fluctuations of scalar and tensor originate on subhorizon scales and they propagate for a long time on superhorizon scales. This can be checked by computing their power spectra which are scale-invariant. However, it would be interesting to check what happens when one computes the power spectra of the massive fluctuations.

4.1 Scalar power spectra

In this section, we first calculate scalar power spectrum. To this end, we consider the conjugate momentum for the field Φ , which is defined by

$$\pi_\Phi = \frac{6a^2}{\kappa} \Phi', \quad (4.4)$$

being obtained from the scalar action (2.25) in the de Sitter background. The canonical quantization is implemented by imposing commutation relation

$$[\hat{\Phi}(\eta, \mathbf{x}), \hat{\pi}_\Phi(\eta, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}') \quad (4.5)$$

with $\hbar = 1$. Now, the operator $\hat{\Phi}$ can be expanded in Fourier modes as

$$\hat{\Phi}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \left(\hat{a}_\mathbf{k} \Phi_\mathbf{k}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + h.c. \right) \quad (4.6)$$

and the operator $\hat{\pi}_\Phi = \frac{6a^2}{\kappa} \Phi'$ is given by (4.6). Substitution of (4.6) and $\hat{\pi}_\Phi$ into (4.5) leads to the commutation relation and Wronskian condition as

$$[\hat{a}_\mathbf{k}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}'), \quad (4.7)$$

$$\Phi_\mathbf{k} \left(\frac{6a^2}{\kappa} \right) (\Phi_\mathbf{k}^*)' - c.c. = i \rightarrow \Phi_\mathbf{k} \frac{d\Phi_\mathbf{k}^*}{dz} - c.c. = -\frac{2i\kappa}{12ka^2}. \quad (4.8)$$

A next step is to choose the initial mode solution to define the Bunch-Davies vacuum $|0\rangle$ when $z \rightarrow \infty$. We note that the solution of $v_\mathbf{k} = a\Phi_\mathbf{k}$ is given to be

$$v_{\mathbf{k}, z \rightarrow \infty} \sim e^{iz}, \quad (4.9)$$

as a solution to the asymptotic scalar equation

$$\left[\frac{d^2}{dz^2} + 1 \right] v_{\mathbf{k}, z \rightarrow \infty}(z) = 0, \quad (4.10)$$

which implies the normalized solution

$$\Phi_{\mathbf{k}, z \rightarrow \infty} \sim \frac{H}{\sqrt{12k^3}} z e^{iz}. \quad (4.11)$$

This is a plane wave to define the Bunch-Davies vacuum. On the other hand, in the super-horizon limit of $z \rightarrow 0$, one has a solution

$$v_{\mathbf{k}, z \rightarrow 0} \sim \frac{1}{z^2}, \quad (4.12)$$

as a solution to

$$\left[\frac{d^2}{dz^2} - \frac{6}{z^2} \right] v_{\mathbf{k}, z \rightarrow 0}(z) = 0. \quad (4.13)$$

It implies that

$$\Phi_{\mathbf{k}, z \rightarrow 0} \sim \frac{1}{z} \quad (4.14)$$

which means that $\Phi_{\mathbf{k}}$ diverges in the superhorizon limit. Then, the power spectrum is given by

$$\mathcal{P}_\Phi = \frac{k^3}{2\pi^2} |\Phi_{\mathbf{k}}|^2 = \frac{H^2}{48\pi} z^3 |e^{i\frac{3\pi}{2}} H_{5/2}^{(1)}(z)|^2. \quad (4.15)$$

In the superhorizon limit of $z \rightarrow 0$, the scalar power spectrum is given by

$$\mathcal{P}_\Phi \Big|_{z \rightarrow 0} \sim \frac{1}{6} \left(\frac{H}{2\pi} \right)^2 \left[\frac{\Gamma(5/2)}{\Gamma(3/2)} \right]^2 \left(\frac{k}{2aH} \right)^{-2} \sim \frac{1}{z^2} \quad (4.16)$$

which blows up in the superhorizon limit of $z \rightarrow 0$.

To obtain the power spectrum for a massive scalar $\delta\phi$, we obtain the conjugate momentum

$$\pi_{\delta\phi} = a^2 \delta\phi' \quad (4.17)$$

which defines the commutator

$$[\delta\hat{\phi}(\eta, \mathbf{x}), \hat{\pi}_{\delta\phi}(\eta, \mathbf{x}')] = i\delta(\mathbf{x}, \mathbf{x}'). \quad (4.18)$$

This implies that the Wronskian condition and commutator are given by

$$a^2 \left(\phi_{\mathbf{k}}(\phi_{\mathbf{k}}^*)' - \phi_{\mathbf{k}}^*(\phi_{\mathbf{k}})' \right) = i, \quad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}'). \quad (4.19)$$

We note that the Wronskian condition together with (3.18) determines the normalized scalar mode

$$\phi_{\mathbf{k}}(z) = \frac{1}{\sqrt{2}} \sqrt{\frac{\pi}{2}} e^{i(\frac{\pi\nu_\phi}{2} + \frac{\pi}{4})} \frac{\sqrt{-\eta}}{a} H_{\nu_\phi}^{(1)}(z), \quad (4.20)$$

where the first factor of $1/\sqrt{2}$ is the normalization from the Wronskian condition (4.19) as $i = 2i \times (1/\sqrt{2})^2$. Then, the power spectrum is defined by

$$\mathcal{P}_{\delta\phi} = \frac{k^3}{2\pi^2} |\phi_{\mathbf{k}}|^2 = \frac{H^2}{8\pi} z^3 |e^{i(\frac{\pi\nu_\phi}{2} + \frac{\pi}{4})} H_{\nu_\phi}^{(1)}(z)|^2. \quad (4.21)$$

In the case of $\nu_\phi = 3/2$ ($m_\phi^2 = 0$), it leads to the power spectrum for a massless scalar as

$$\mathcal{P}_{\delta\phi} \Big|_{m_\phi^2 \rightarrow 0, z \rightarrow 0} \sim \left(\frac{H}{2\pi} \right)^2 \quad (4.22)$$

which is a scale-invariant spectrum.

In the limit of $z \rightarrow 0$, one refers to the form (3.49) which implies that the superhorizon limit of the inflaton power spectrum is given by

$$\mathcal{P}_{\delta\phi} \Big|_{z \rightarrow 0} \sim \left(\frac{H}{2\pi} \right)^2 \left[\frac{\Gamma(\nu_\phi)}{\Gamma(3/2)} \right]^2 \left(\frac{k}{2aH} \right)^{3-2\nu_\phi}. \quad (4.23)$$

Assuming $m_\phi^2/H^2 \ll 1$ such that $\nu_\phi \simeq 3/2 - m_\phi^2/(3H^2) + \mathcal{O}(m_\phi^4/H^4)$, one has

$$\mathcal{P}_{\delta\phi} \simeq \left(\frac{H}{2\pi} \right)^2 \left(\frac{k}{2aH} \right)^{\frac{2m_\phi^2}{3H^2}} \quad (4.24)$$

whose spectral index takes the form

$$n_{\delta\phi} - 1 = \frac{d \ln \mathcal{P}_{\delta\phi}}{d \ln k} \simeq \frac{2m_\phi^2}{3H^2}. \quad (4.25)$$

In the case of a finite mass, the spectrum would be slightly blue due to the massive nature. However, for $m_\phi^2 \ll H^2$, the spectrum is almost scale-invariant and the condition of $m_\phi^2 \ll H^2$ determines a long-lasting de Sitter inflation.

Since a longitudinally light massive vector \mathcal{A}^\parallel satisfies the massive scalar equation (3.15), we expect to have its power spectrum as

$$\mathcal{P}_{\mathcal{A}^\parallel} = \frac{k^3}{2\pi^2} |\mathcal{A}_{\mathbf{k}}^\parallel|^2 = \frac{H^2}{8\pi} z^3 |e^{i(\frac{\pi\nu}{2} + \frac{\pi}{4})} H_{\nu_{\mathcal{A}^\parallel}}^{(1)}(z)|^2 \quad (4.26)$$

with

$$\nu_{\mathcal{A}^\parallel} = \sqrt{\frac{9}{4} - \frac{m_F^2}{H^2}}. \quad (4.27)$$

Considering $m_F^2/H^2 \ll 1$ such that $\nu_F \simeq 3/2 - m_F^2/(3H^2) + \mathcal{O}(m_F^4/H^4)$, we have

$$\mathcal{P}_{\mathcal{A}^\parallel} \simeq \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{2aH}\right)^{\frac{2m_F^2}{3H^2}} \quad (4.28)$$

whose spectral index takes the form [21]

$$n_{\mathcal{A}^\parallel} - 1 = \frac{d \ln \mathcal{P}_{\mathcal{A}^\parallel}}{d \ln k} \simeq \frac{2m_F^2}{3H^2}. \quad (4.29)$$

4.2 Vector power spectrum

We now calculate vector power spectrum. For this purpose, we define a commutation relation for the vector. In the bilinear action (2.20), the conjugate momentum for the field Ψ_j is defined by

$$\pi_\Psi^j = \frac{1}{2\kappa m^2} \Psi^{j'}. \quad (4.30)$$

The canonical quantization is implemented by imposing the commutation relation

$$[\hat{\Psi}_j(\eta, \mathbf{x}), \hat{\pi}_\Psi^j(\eta, \mathbf{x}')] = 2i\delta(\mathbf{x} - \mathbf{x}') \quad (4.31)$$

with $\hbar = 1$.

Now, the operator $\hat{\Psi}_j$ can be expanded in Fourier modes as

$$\hat{\Psi}_j(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \sum_{s=R,L} \left(\tilde{p}_j^s(\mathbf{k}) \hat{a}_{\mathbf{k}}^s \Psi_{\mathbf{k}}^s(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + h.c. \right) \quad (4.32)$$

and the operator $\hat{\pi}_\Psi^j = \frac{1}{2\kappa m^2} \hat{\Psi}^{j'}$ can be easily obtained from (4.32). Here the circularly polarized vector \tilde{p}_j^L satisfies $\tilde{p}_j^L(\tilde{p}^{jL})^* = \tilde{p}_j^R(\tilde{p}^{jR})^* = 1$ and the superscript s in (4.32) denotes (L, R) circularly polarized vector.

Plugging (4.32) and $\hat{\pi}_\Psi^j$ into (4.31), we find the commutation relation and Wronskian condition as

$$[\hat{a}_{\mathbf{k}}^s, \hat{a}_{\mathbf{k}'}^{s'\dagger}] = \delta^{ss'} \delta(\mathbf{k} - \mathbf{k}'), \quad (4.33)$$

$$(4.34)$$

$$\Psi_{\mathbf{k}}^s \left(-\frac{2}{\kappa m^2} \right) (\Psi_{\mathbf{k}}^{*s})' - c.c. = -4i \rightarrow \Psi_{\mathbf{k}}^s \frac{d\Psi_{\mathbf{k}}^{*s}}{dz} - c.c. = -\frac{2i\kappa m^2}{k}. \quad (4.35)$$

We choose the initial mode solution for a Bunch-Davies vacuum $|0\rangle$

$$\Psi_{\mathbf{k}, z \rightarrow \infty}^s \sim \sqrt{\frac{\kappa m^2}{k}} \underbrace{e^{iz}} = \sqrt{\frac{\kappa m^2}{k}} \underbrace{\sqrt{\frac{\pi}{2}} e^{i(\frac{\pi \nu_s}{2} + \frac{\pi}{4})} \sqrt{z} H_{\nu_s}^{(1)}(z \rightarrow \infty)} \quad (4.36)$$

which is obtained as a solution to the asymptotic vector equation

$$\left[\frac{d^2}{dz^2} + 1 \right] \Psi_{\mathbf{k}, z \rightarrow \infty}^s(z) = 0 \quad (4.37)$$

together with the Wronskian condition (4.35). In this case, Eq. (3.20) can be written as

$$\left[\frac{d^2}{dz^2} + 1 + \frac{\tilde{m}_s^2}{H^2} \frac{1}{z^2} \right] \Psi_{\mathbf{k}}^s(z) = 0. \quad (4.38)$$

for $\theta = (M_{\text{P}}^3 \eta)^{-1}$. The full solution to (4.38) is given by the Hankel function

$$\Psi_{\mathbf{k}}^s = \sqrt{\frac{\kappa m^2}{k}} \sqrt{\frac{\pi}{2}} e^{i(\frac{\pi \nu_s}{2} + \frac{\pi}{4})} \sqrt{z} H_{\nu_s}^{(1)}(z), \quad (4.39)$$

where $\nu_s < 1/2$ is given by (3.45). On the other hand, the vector power spectrum is defined by

$$\langle 0 | \hat{\Psi}_j(\eta, \mathbf{x}) \hat{\Psi}^j(\eta, \mathbf{x}) | 0 \rangle = \int d^3 \mathbf{k} \frac{\mathcal{P}_{\Psi}}{4\pi k^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}, \quad (4.40)$$

where we used the Bunch-Davies vacuum state imposing $\hat{a}_{\mathbf{k}}^s | 0 \rangle = 0$ and a quantity \mathcal{P}_{Ψ} in (4.40) denotes $\mathcal{P}_{\Psi} \equiv \sum_{s=R,L} \frac{k^3}{2\pi^2} \left| \Psi_{\mathbf{k}}^s \right|^2$. Plugging (4.39) into (4.40), we find the scale-dependent spectrum

$$\mathcal{P}_{\Psi} = \sum_{s=R,L} \frac{k^2 m^2}{4\pi M_{\text{P}}^2} \left(z \left| e^{i(\frac{\pi \nu_s}{2} + \frac{\pi}{4})} H_{\nu_s}^{(1)}(z) \right|^2 \right). \quad (4.41)$$

On the other hand, in the limit of $z \rightarrow 0$, one refers to the form of Hankel function

$$H_{\nu_s}^{(1)}(z \rightarrow 0) \sim \frac{i}{\pi} \frac{\Gamma(\nu_s)}{\left(\frac{1}{2}z\right)^{\nu_s}} \Big|_{z \rightarrow 0}, \quad (4.42)$$

which implies the superhorizon limit of the vector power spectrum

$$\mathcal{P}_{\Psi} \Big|_{z \rightarrow 0} = \sum_{s=R,L} \frac{1}{2} \left(\frac{2aH}{\pi} \right)^2 \left(\frac{m}{M_{\text{P}}} \right)^2 \left(\frac{\Gamma(\nu_s)}{\Gamma(1/2)} \right)^2 \left(\frac{k}{2aH} \right)^{3-2\nu_s}. \quad (4.43)$$

For $\nu_s = 1/2$ ($m^2 = 0$), the power spectrum of the vector perturbation is also zero as

$$\mathcal{P}_{\Psi} \Big|_{m^2 \rightarrow 0, z \rightarrow 0} = 0. \quad (4.44)$$

We wish to explain why \mathcal{P}_{Ψ} approaches zero in the limits of $m^2 \rightarrow 0$ and $z \rightarrow 0$. In the case of $m^2 \rightarrow 0$, the vector field becomes conformally invariant as shown in (2.20) and thus, it is

not gravitationally produced because it does not couple to the expanding gravitational (de Sitter) background [21].

In the case of $\kappa m^2 = (m/M_P)^2 = 1$, the Weyl-squared term in (2.20) reproduces a transversely massive vector Lagrangian in (3.25). For $\theta = 0$, thus, one recovers the power spectrum $\mathcal{P}_{\mathcal{A}^\perp}$ for a transversely massive vector [21]

$$\mathcal{P}_\Psi \Big|_{\theta \rightarrow 0, z \rightarrow 0} = \left(\frac{m}{M_P} \right)^2 \mathcal{P}_{\mathcal{A}^\perp}, \quad (4.45)$$

where

$$\mathcal{P}_{\mathcal{A}^\perp} = \left(\frac{2aH}{\pi} \right)^2 \left(\frac{\Gamma(\nu_{\mathcal{A}^\perp})}{\Gamma(1/2)} \right)^2 \left(\frac{k}{2aH} \right)^{3-2\nu_{\mathcal{A}^\perp}} \quad (4.46)$$

with

$$\nu_{\mathcal{A}^\perp} = \sqrt{\frac{1}{4} - \frac{m_F^2}{H^2}}. \quad (4.47)$$

If one defines a physical vector $V_i = \mathcal{A}^\perp/a$, then its power spectrum takes the form

$$\mathcal{P}_V = \left(\frac{2H}{\pi} \right)^2 \left(\frac{\Gamma(\nu_{\mathcal{A}^\perp})}{\Gamma(3/2)} \right)^2 \left(\frac{k}{2aH} \right)^{3-2\nu_{\mathcal{A}^\perp}} \quad (4.48)$$

which still vanishes in the limit of $z \rightarrow 0$ ($k \ll aH$) and for $\nu_{\mathcal{A}^\perp} < 1/2$. Its spectral index takes the form

$$n_V = \frac{d \ln \mathcal{P}_V}{d \ln k} \simeq 2 + \frac{2m_F^2}{H^2}. \quad (4.49)$$

for $m_F^2 \ll H^2$. Even for $m_F^2 \ll H^2$, the spectrum is not scale-invariant.

4.3 Tensor power spectrum

In order to derive power spectrum for tensor perturbations in the ECSW gravity, we first rewrite the fourth-order bilinear action (2.21) by using the Ostrogradsky formalism as

$$\begin{aligned} \kappa S_{\text{ECSW}}^{(\text{TO})} &\equiv \int d^4x \mathcal{L}_{\text{ECSW}}^{\text{O}} \\ &= \frac{1}{8} \int d^4x \left[a^2 (\alpha_{ij} \alpha^{ij} - \partial_k h_{ij} \partial^k h^{ij}) - \theta' \epsilon_i^{jk} (\alpha^{\ell i} \partial_j \alpha_{k\ell} - \partial^\ell h^{pi} \partial_j \partial_\ell h_{kp}) \right. \\ &\quad \left. - \frac{1}{m^2} (\alpha'_{ij} \alpha'^{ij} - 2\partial_k \alpha_{ij} \partial^k \alpha^{ij} + \partial^2 h_{ij} \partial^2 h^{ij}) + 2\beta^{ij} (\alpha_{ij} - h'_{ij}) \right], \end{aligned} \quad (4.50)$$

where α_{ij} is a new variable and β^{ij} is a Lagrange multiplier. The action (4.50) is surely the second-order bilinear action which shows that the Ostrogradsky formalism is a well-known method to handle a higher-order action [22–24]. Especially, the quantization and mean square expectation value of the nondegenerate Pais-Uhlenbeck oscillator will be implemented to obtain the tensor power spectrum [23]. We have two tensors h_{ij} and α_{ij} that amount to 4(=2+2) DOF which explains why the fourth-order action (2.21) describes 4 DOF. From $\mathcal{L}_{\text{ECSW}}^{\text{O}}$ in (4.50), the conjugate momenta are given by

$$\pi_h^{ij} = -\frac{1}{4\kappa} \beta^{ij}, \quad \pi_\alpha^{ij} = -\frac{1}{4\kappa m^2} \alpha'^{ij}. \quad (4.51)$$

The equation of motion could be obtained by varying the action (4.50) with respect to β^{ij} and α_{ij} as

$$\alpha_{ij} = h'_{ij}, \quad \beta_{ij} = -a^2 \alpha_{ij} - \theta' \epsilon_k^{\ell i} \partial_\ell \alpha^{jk} - \frac{1}{m^2} (\alpha''_{ij} - 2\partial^2 \alpha_{ij}). \quad (4.52)$$

Plugging (4.52) into (4.51), the conjugate momenta are given by

$$\pi_h^{ij} = \frac{1}{4\kappa} \left(a^2 h'^{ij} - \frac{2}{m^2} \partial^2 h'^{ij} + \frac{1}{m^2} h'''^{ij} + \theta' \epsilon_k^{\ell i} \partial_\ell h'^{jk} \right), \quad \pi_{h'}^{ij} = -\frac{1}{4\kappa m^2} h''^{ij}, \quad (4.53)$$

which are the same forms obtained from the variation of (2.21) with respect to h'^{ij} and h''^{ij} . Then, the corresponding Hamiltonian is given by

$$\mathcal{H}_{\text{ECSW}} = \pi_h^{ij} h'_{ij} + \pi_{h'}^{ij} h''_{ij} - \tilde{\mathcal{L}}_{\text{ECSW}} \quad (4.54)$$

with $S_{\text{ECSW}}^{(T)} = \int d^4x \tilde{\mathcal{L}}_{\text{ECSW}}$ in (2.21).

At this stage, we would like to mention how to take the limit of $m^2 \rightarrow 0$ in (4.50). In this case, the above relations (4.51) and (4.52) get lost and h_{ij} becomes non-dynamical. However, taking a $m^2 \rightarrow 0$ limit of the original tensor action (2.21) leads to the purely Weyl-squared term. In the auxiliary Ostrogradsky formalism, we could not take the $m^2 \rightarrow 0$ limit directly. As was emphasized in Ref. [23], there is a crucial difference between the general Hamiltonian $\mathcal{H}_{\text{ECSW}}^{\text{O}} = \pi_h^{ij} h'_{ij} + \pi_\alpha^{ij} \alpha'_{ij} - \mathcal{L}_{\text{ECSW}}^{\text{O}}$ and $\mathcal{H}_{\text{ECSW}}$ (4.54). The latter Hamiltonian was obtained when one imposes equation (4.52) and uses (4.53), and it is thus called the classical and stationary Ostrogradsky Hamiltonian which corresponds to the fourth-order bilinear action (2.21). Hence, taking the limit of $m^2 \rightarrow 0$ in (4.50) must be done after imposing (4.52) to derive the Weyl-squared term whose gravitational waves are not gravitationally amplified because it decoupled from the de Sitter background. This explains why one takes the $m^2 \rightarrow 0$ limit in (2.21) to derive a conformally invariant Weyl-squared term.

Now, the canonical quantization is accomplished by imposing equal-time commutation relations:

$$[\hat{h}_{ij}(\eta, \mathbf{x}), \hat{\pi}_h^{ij}(\eta, \mathbf{x}')] = 2i\delta(\mathbf{x} - \mathbf{x}'), \quad [\hat{h}'_{ij}(\eta, \mathbf{x}), \hat{\pi}_{h'}^{ij}(\eta, \mathbf{x}')] = 2i\delta(\mathbf{x} - \mathbf{x}'), \quad (4.55)$$

where the factor 2 is coming from the fact that h_{ij} and $h'_{ij}(= \alpha_{ij})$ represent 2 DOF, respectively.

Here we will employ h_{ij} only to compute the tensor power spectrum by introducing two-types of mode solutions $h_{\mathbf{k}}^{s(1)}$ and $h_{\mathbf{k}}^{s(2)}$ which correspond to Einstein and Chern-Simons-Weyl tensor modes. Its operator \hat{h}_{ij} can be expanded in Fourier modes as

$$\hat{h}_{ij}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \left[\sum_{s=R,L} \left(\tilde{p}_{ij}^s(\mathbf{k}) \hat{a}_{\mathbf{k}}^s h_{\mathbf{k}}^{s(1)}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \tilde{p}_{ij}^s(\mathbf{k}) \hat{b}_{\mathbf{k}}^s h_{\mathbf{k}}^{s(2)}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \right) + h.c. \right] \quad (4.56)$$

Also, we find from (4.56) that the operator $\hat{\pi}_h^{ij}$ is given by

$$\begin{aligned} \hat{\pi}_h^{ij}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{4\kappa} \left[\sum_{s=R,L} \left(\tilde{p}_{ij}^s(\mathbf{k}) \hat{a}_{\mathbf{k}}^s \left\{ \xi^s \left(h_{\mathbf{k}}^{s(1)}(\eta) \right)' + \frac{1}{m^2} \left(h_{\mathbf{k}}^{s(1)}(\eta) \right)''' \right\} e^{i\mathbf{k}\cdot\mathbf{x}} \right. \right. \\ \left. \left. + \tilde{p}_{ij}^s(\mathbf{k}) \hat{b}_{\mathbf{k}}^s \left\{ \xi^s \left(h_{\mathbf{k}}^{s(2)}(\eta) \right)' + \frac{1}{m^2} \left(h_{\mathbf{k}}^{s(2)}(\eta) \right)''' \right\} e^{i\mathbf{k}\cdot\mathbf{x}} \right) + h.c. \right], \end{aligned} \quad (4.57)$$

where ξ^s is

$$\xi^s = a^2 + \frac{2}{m^2} k^2 - \lambda^s k \theta'. \quad (4.58)$$

On the other hand, h'_{ij} is given by

$$\begin{aligned} \hat{h}'_{ij}(\eta, \mathbf{x}) = & \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \left[\sum_{s=R,L} \left(\tilde{p}_{ij}^s(\mathbf{k}) \hat{a}_{\mathbf{k}}^s \left(h_{\mathbf{k}}^{s(1)}(\eta) \right)' e^{i\mathbf{k}\cdot\mathbf{x}} \right. \right. \\ & \left. \left. + \tilde{p}_{ij}^s(\mathbf{k}) \hat{b}_{\mathbf{k}}^s \left(h_{\mathbf{k}}^{s(2)}(\eta) \right)' e^{i\mathbf{k}\cdot\mathbf{x}} \right) + h.c. \right], \end{aligned} \quad (4.59)$$

and $\pi_{h'ij}$ takes the form

$$\begin{aligned} \hat{\pi}_{h'ij}(\eta, \mathbf{x}) = & -\frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{4\kappa m^2} \left[\sum_{s=R,L} \left(\tilde{p}_{ij}^s(\mathbf{k}) \hat{a}_{\mathbf{k}}^s \left(h_{\mathbf{k}}^{s(1)}(\eta) \right)'' e^{i\mathbf{k}\cdot\mathbf{x}} \right. \right. \\ & \left. \left. + \tilde{p}_{ij}^s(\mathbf{k}) \hat{b}_{\mathbf{k}}^s \left(h_{\mathbf{k}}^{s(2)}(\eta) \right)'' e^{i\mathbf{k}\cdot\mathbf{x}} \right) + h.c. \right]. \end{aligned} \quad (4.60)$$

Substituting (4.56), (4.57), (4.59), and (4.60) into (4.55) leads to the consistent commutation relations and Wronskian conditions:

$$[\hat{a}_{\mathbf{k}}^s, \hat{a}_{\mathbf{k}'}^{s'\dagger}] = \delta^{ss'} \delta(\mathbf{k} - \mathbf{k}'), \quad [\hat{b}_{\mathbf{k}}^s, \hat{b}_{\mathbf{k}'}^{s'\dagger}] = -\delta^{ss'} \delta(\mathbf{k} - \mathbf{k}'), \quad (4.61)$$

$$\left[h_{\mathbf{k}}^{s(1)} \left\{ \xi^s(h_{\mathbf{k}}^{*s(1)})' + \frac{1}{m^2} (h_{\mathbf{k}}^{*s(1)})''' \right\} - h_{\mathbf{k}}^{s(2)} \left\{ \xi^s(h_{\mathbf{k}}^{*s(2)})' + \frac{1}{m^2} (h_{\mathbf{k}}^{*s(2)})''' \right\} \right] - c.c. = 4i\kappa, \quad (4.62)$$

$$\left[(h_{\mathbf{k}}^{s(1)})' (h_{\mathbf{k}}^{*s(1)})'' - (h_{\mathbf{k}}^{s(2)})' (h_{\mathbf{k}}^{*s(2)})'' \right] - c.c. = -4i\kappa m^2, \quad (4.63)$$

where $\tilde{p}_{ij}^L(\tilde{p}^{ijL})^* = \tilde{p}_{ij}^R(\tilde{p}^{ijR})^* = 1$ and a superscript “s” in (4.62) and (4.63) does not denote summation over (L, R). We note that two mode operators ($\hat{a}_{\mathbf{k}}^s, \hat{b}_{\mathbf{k}}^s$) are needed to take into account the fourth-order bilinear tensor action (4.50) by using the Ostrogradsky formalism.

Importantly, introducing $\mu_{\mathbf{k}}^{s(E)} = \mu_{\mathbf{k}}^{s(1)}$ and $\mu_{\mathbf{k}}^{s(CSW)} = \mu_{\mathbf{k}}^{s(2)}$, Eqs.(3.37) and (3.38) in the de Sitter background with $z = -\eta k$ can be written by (3.39) and (3.40), respectively. The consistency condition to satisfy Eqs. (3.39)-(3.40) is given by

$$h_{\mathbf{k}}^{s(1)} \frac{dh_{\mathbf{k}}^{*s(1)}}{dz} - c.c. = h_{\mathbf{k}}^{s(2)} \frac{dh_{\mathbf{k}}^{*s(2)}}{dz} - c.c. = -\frac{4i\kappa m^2 z^2}{k^3(2 + \tilde{m}_s^2/H^2)}, \quad (4.64)$$

where \tilde{m}_s^2 is given by $\tilde{m}_{R/L}^2 = m^2(1 \pm kH^2 M_P^{-3})$. We note that if two modes have the same normalization, we could not determine their normalization constants because Eqs.(4.62) and (4.63) provides the same relation.

It turns out that when we consider the initial condition to set the Bunch-Davies vacuum, the two solutions are given by

$$h_{\mathbf{k}}^{s(1)} = \sqrt{\frac{2\kappa m^2}{k^3(2 + \tilde{m}_s^2/H^2)}} \sqrt{\frac{\pi}{2}} e^{i\pi z^{\frac{3}{2}}} H_{3/2}^{(1)}(z), \quad (4.65)$$

$$h_{\mathbf{k}}^{s(2)} = \sqrt{\frac{2\kappa m^2}{k^3(2 + \tilde{m}_s^2/H^2)}} \sqrt{\frac{\pi}{2}} e^{i(\frac{\pi\nu_s}{2} + \frac{\pi}{4})} z^{\frac{3}{2}} H_{\nu_s}^{(1)}(z), \quad (4.66)$$

where $H_{\nu_s}^{(1)}$ is the Hankel function and here ν_s is given by (3.45). Comparing these solutions with (3.48), the normalizations are fixed in the former cases.

On the other hand, the power spectrum of the gravitational waves is defined by

$$\langle 0 | \hat{h}_{ij}(\eta, \mathbf{x}) \hat{h}^{ij}(\eta, \mathbf{x}') | 0 \rangle = \int d^3\mathbf{k} \frac{\mathcal{P}_T}{4\pi k^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}, \quad (4.67)$$

where we choose the Bunch-Davies vacuum state $|0\rangle$ by imposing $\hat{a}_{\mathbf{k}}^s|0\rangle = 0$ and $\hat{b}_{\mathbf{k}}^s|0\rangle = 0$. A quantity \mathcal{P}_T in (4.67) denotes $\mathcal{P}_T \equiv \sum_{s=R,L} \mathcal{P}^s$ given as

$$\mathcal{P}^s = \frac{k^3}{2\pi^2} \left(\left| h_{\mathbf{k}}^{s(1)} \right|^2 - \left| h_{\mathbf{k}}^{s(2)} \right|^2 \right), \quad (4.68)$$

where the difference $(-)$ arises from the commutation relations (4.61).

It is very important to note that the tensor power spectrum (4.68) is based on the factorization (3.34) and (3.35) which can be realized only when θ takes the form $\theta = c/\eta$. Unless choosing this θ , it is difficult to obtain the corresponding tensor power spectrum. Substituting (4.65) and (4.66) into (4.67), we obtain with $\kappa = 1/M_P^2$

$$\mathcal{P}_T = \sum_{s=R,L} \frac{H^2}{\pi^2 M_P^2} \frac{m^2}{\tilde{m}_s^2 + 2H^2} \left(1 + z^2 - \frac{\pi}{2} z^3 |e^{i(\frac{\pi\nu_s}{2} + \frac{\pi}{4})} H_{\nu_s}^{(1)}(z)|^2 \right). \quad (4.69)$$

In the limit of $m^2 \rightarrow 0$ (keeping Weyl-squared term only), one has $\nu_s \rightarrow 1/2$ and $\tilde{m}^2 \rightarrow 0$. This case provides the zero power spectrum as

$$\mathcal{P}_T \Big|_{m^2 \rightarrow 0} \rightarrow 0 \quad (4.70)$$

which implies that the tensor perturbation becomes conformally invariant and thus, they are not gravitationally produced because they are decoupled from the expanding de Sitter background as was shown in (2.21).

In the case of Einstein-Weyl gravity with $\theta = 0$ ($\tilde{m}_s^2 = m^2$), the tensor power spectrum (4.69) reduces to

$$\mathcal{P}_T^{\text{EW}} = \mathcal{P}^{\text{GW}} \frac{m^2}{m^2 + 2H^2} \left(1 + z^2 - \frac{\pi}{2} z^3 |e^{i(\frac{\pi\nu}{2} + \frac{\pi}{4})} H_{\nu}^{(1)}(z)|^2 \right), \quad (4.71)$$

where $\mathcal{P}^{\text{GW}} = \frac{2H^2}{\pi^2 M_P^2}$ is the power spectrum for gravitational waves and $\nu = \sqrt{1/4 - m^2/H^2}$. This is the same power spectrum obtained in Ref.[25].

In the superhorizon limit of $z \rightarrow 0$, the second in (4.69) is zero and the third term approaches zero as

$$\frac{\pi}{2} z^3 |e^{i(\frac{\pi\nu_s}{2} + \frac{\pi}{4})} H_{\nu_s}^{(1)}|^2 \Big|_{z \rightarrow 0} = \frac{1}{2\pi} \Gamma(\nu_s)^2 2^{2\nu_s} z^{3-2\nu_s} \Big|_{z \rightarrow 0} \rightarrow 0 \quad (4.72)$$

for $\nu_s < 1/2$ which is confirmed from (3.45). In this case, (4.69) leads to the power spectrum for GWs

$$\begin{aligned} \mathcal{P}_T \Big|_{z \rightarrow 0} &= \sum_{s=R,L} \frac{H^2}{\pi^2 M_P^2} \left[\frac{m^2}{\tilde{m}_s^2 + 2H^2} \right] \\ &= \mathcal{P}^{\text{GW}} \frac{(1 + 2H^2/m^2)}{(1 + 2H^2/m^2)^2 - (kH^2/M_P^3)^2} \equiv \mathcal{P}^{\text{GW}} \Xi_{\text{CSW}}, \end{aligned} \quad (4.73)$$

where one recovers $\Xi_{\text{EW}} = \frac{1}{1+2H^2/m^2}$ in $\mathcal{P}_{\text{T}}^{\text{EW}}|_{z \rightarrow 0} = \mathcal{P}^{\text{GW}} \Xi_{\text{EW}}$ of the Einstein-Weyl gravity [25]. We note that $\mathcal{P}_{\text{T}}|_{z \rightarrow 0}$ is not a scale-invariant spectrum, but $\mathcal{P}_{\text{T}}^{\text{EW}}|_{z \rightarrow 0}$ is a scale-invariant spectrum. Considering the bound (3.46), we have $2H^2/m^2 > 8$. Also, we have $kH^2/M_{\text{P}}^3 \ll 1$. In this case, the mass-squared term damps out smoothly the spectrum of primordial gravitational waves (\mathcal{P}^{GW}) because

$$\mathcal{P}_{\text{T}}|_{z \rightarrow 0} < \frac{1}{9} \mathcal{P}^{\text{GW}}. \quad (4.74)$$

Finally, we would like to mention the massive tensor equation from the general massive gravity [26]

$$h''_{ij} + 2\mathcal{H}h'_{ij} + m_2^2 a^2 h_{ij} - \partial^2 h_{ij} = 0, \quad (4.75)$$

which is the same equation (3.7) as for the massive scalar $\delta\phi$. Considering the normalization of $\delta\phi \rightarrow \frac{M_{\text{P}}}{2} h_{ij}$, the power spectrum is given by

$$\mathcal{P}_{\text{GMG}} = 2 \times \left(\frac{2}{M_{\text{P}}} \right)^2 \mathcal{P}_{\delta\phi} \Big|_{\nu_\phi \rightarrow \nu_{m_2}} = \frac{H^2}{\pi M_{\text{P}}^2} z^3 |e^{i(\frac{\pi\nu_{m_2}}{2} + \frac{\pi}{4})} H_{\nu_{m_2}}^{(1)}(z)|^2, \nu_{m_2} = \sqrt{\frac{9}{4} - \frac{m_2^2}{H^2}} \quad (4.76)$$

with $\mathcal{P}_{\delta\phi}$ (4.21). In the limit of $z \rightarrow 0$ and $m_2^2/H^2 \ll 1$, one has the massive tensor power spectrum

$$\mathcal{P}_{\text{GMG}} \simeq \left(\frac{2H^2}{\pi^2 M_{\text{P}}^2} \right) \left(\frac{k}{2aH} \right)^{\frac{2m_2^2}{3H^2}} \quad (4.77)$$

whose spectral index takes the form

$$n_{\text{GMG}} = \frac{d \ln \mathcal{P}_{\text{GMG}}}{d \ln k} \simeq \frac{2m_2^2}{3H^2}. \quad (4.78)$$

In the case of $m_2^2 = 0$, we have the power spectrum for GWs as and its spectral index

$$\mathcal{P}_{\text{GMG}} \Big|_{m_2^2 \rightarrow 0} = \left(\frac{2H^2}{\pi^2 M_{\text{P}}^2} \right), \quad n_{\text{GMG}} \Big|_{m_2^2 \rightarrow 0} = 0. \quad (4.79)$$

5 Discussions

In the Einstein gravity, all fluctuations of scalar and tensor originate on subhorizon scales and they propagate for a long time on superhorizon scales. There is no vector propagation. This can be checked by computing their power spectra in the superhorizon limit of $z \rightarrow 0$. However, we have found the different results for massive fluctuations composed of scalar $\Phi = \Psi$, vector Ψ_i with 2 DOF, and tensor h_{ij} with 4 DOF.

First of all, we have derived a power spectrum \mathcal{P}_Φ for the scalar Φ which blows up in the superhorizon limit. In the case of Einstein gravity, however, Φ was combined to give comoving curvature perturbation \mathcal{R} on the slow-roll inflation. Also, we have obtained a scale-variant power spectrum $\mathcal{P}_{\delta\phi}$ for a massive scalar $\delta\phi$.

The power spectra of massive vector Ψ_i shows clearly that their fluctuations do not propagate for a long time on superhorizon scales. It decays quickly in the superhorizon limit of $z \rightarrow 0$. Also, in the limit of $m^2 \rightarrow 0$ (keeping the Weyl-squared term only), it disappears because the vector field becomes conformally invariant as shown in (2.20). It is not gravitationally produced because it does not couple to the expanding gravitational

(de Sitter) background [21]. There exist close connection in power spectra between $\{\Psi_i, \delta\phi\}$ and $\{\mathcal{A}^\perp, \mathcal{A}^\parallel\}$, where $\delta\phi$ a massive scalar, \mathcal{A}^\perp and \mathcal{A}^\parallel are a transversely massive vector and longitudinally light massive vector, respectively [21]. We have found that $\mathcal{P}_\Psi \simeq \mathcal{P}_{\mathcal{A}^\perp}$ and $\mathcal{P}_{\delta\phi} \simeq \mathcal{P}_{\mathcal{A}^\parallel}$. In order to compute \mathcal{P}_Ψ , however, we have chosen the conventional conjugate momentum $\pi_\Psi = \frac{1}{2\kappa m^2}(\Psi^j)'$ instead of $\tilde{\pi}_\Psi = \frac{1}{2\kappa m^2}\partial^2(\Psi^j)'$ obtained from (2.20). The latter induces an inconsistent quantization of $[\hat{a}_{\mathbf{k}}^s, \hat{a}_{\mathbf{k}'}^{s'}] = -\delta^{ss'}\delta(\mathbf{k}-\mathbf{k}')$ and thus, an unconventional power spectrum.

The power spectrum of tensor has taken a complicated form (4.69) because it was obtained by using the Ostrogradsky formalism to handle the fourth-order theory. In this case, one usually introduces two sets of lowering and raising operator for all \mathbf{k} . Here, the ghost state problem might appear because the tensor equation is a fourth-order equation with respect to conformal time η . In the superhorizon limit of $z \rightarrow 0$, however, the tensor power spectrum coming from massive GWs (2 DOF) decays quickly, remaining the conventional tensor (2 DOF) power spectrum. Importantly, in the limit of $m^2 \rightarrow 0$ (keeping the Weyl-squared term only), the tensor power spectrum (4.69) disappears because the tensor field becomes conformally invariant as shown in (2.21). It is not gravitationally produced because it is decoupled from the expanding gravitational (de Sitter) background. However, considering the power spectrum (4.76) from the general massive gravity, its massless limit (4.79) recovers the conventional tensor power spectrum and spectral index. This implies that a different massive gravity provides a different power spectrum.

Finally, we would like to mention the choice of Chern-Simons scalar θ . It remains undetermined in the de Sitter inflation. For $\theta = c \ln \eta$, Eq.(3.32) could describe propagation of circularly polarized GWs because the Chern-Simons term is regarded as a small correction. However, their power spectrum is the same as that of GWs on the de Sitter inflation [19]. For $\theta = c/\eta$, however, one make factorization of fourth-order tensor equation which is a necessary step to compute the tensor power spectrum. In this work, we have chosen $\theta = c/\eta$ to obtain two second-order tensor equations (3.39) and (3.40) because $\theta = c \ln \eta$ unlikely factorize the fourth-order equation into two second-order equations.

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Appendix: Factorization of the fourth-order tensor equation

In the de Sitter background with $z = -\eta k$, the fourth-order differential equation (3.27) can be rewritten as

$$\begin{aligned} \frac{d^4}{dz^4} h_{\mathbf{k}}^s(z) + 2 \frac{d^2}{dz^2} h_{\mathbf{k}}^s(z) + h_{\mathbf{k}}^s(z) + m^2 \left\{ \left(\frac{1}{H^2 z^2} + \lambda^s \frac{d\theta}{dz} \right) \frac{d^2}{dz^2} h_{\mathbf{k}}^s(z) \right. \\ \left. - \left(\frac{2}{H^2 z^3} - \lambda^s \frac{d^2 \theta}{dz^2} \right) \frac{d}{dz} h_{\mathbf{k}}^s(z) + \left(\frac{1}{H^2 z^2} + \lambda^s \frac{d\theta}{dz} \right) h_{\mathbf{k}}^s(z) \right\} = 0. \end{aligned} \quad (5.1)$$

To factorize the above fourth-order equation (5.1) into two second-order equations, we consider a factorized form

$$\left(X_1(z) \frac{d^2}{dz^2} + X_2(z) \frac{d}{dz} + X_3(z) \right) \left(X_4(z) \frac{d^2}{dz^2} + X_5(z) \frac{d}{dz} + X_6(z) \right) h_{\mathbf{k}}^s(z) = 0. \quad (5.2)$$

Comparing the coefficients of $\frac{d^2}{dz^2} h_{\mathbf{k}}^s(z)$, $\frac{d^3}{dz^3} h_{\mathbf{k}}^s(z)$, $\frac{d^4}{dz^4} h_{\mathbf{k}}^s(z)$ in (5.1) and (5.2), we find

$$X_1 = \frac{1}{X_4}, \quad X_2 = \frac{2}{z X_4} - \frac{2}{X_4^2} \frac{dX_4}{dz}, \quad (5.3)$$

$$X_3 = \frac{1}{X_4} \left\{ 2 - \frac{X_6}{X_4} - \frac{2}{z X_4} \frac{dX_4}{dz} + \frac{2}{X_4^2} \left(\frac{dX_4}{dz} \right)^2 - \frac{1}{X_4} \frac{d^2 X_4}{dz^2} + m^2 \left(\frac{1}{H^2 z^2} + \lambda^s \frac{d\theta}{dz} \right) \right\}, \quad (5.4)$$

where we used a condition of $X_5 = -2X_4/z$, which yields the Schrödinger-type equation for $\mu_{\mathbf{k}}^s (= h_{\mathbf{k}}^s/z)$. Substituting Eqs. (5.3) and (5.4) into (5.2) and comparing the coefficients of $h_{\mathbf{k}}^s(z)$ and $\frac{d}{dz} h_{\mathbf{k}}^s(z)$ in (5.1) and (5.2) leads to

$$\begin{aligned} 1 + \frac{m^2}{H^2 z^2} + m^2 \lambda^s \frac{d\theta}{dz} - \left(2 + \frac{m^2}{H^2 z^2} + m^2 \lambda^s \frac{d\theta}{dz} \right) \frac{X_6}{X_4} + \frac{X_6^2}{X_4^2} \\ - \frac{2}{z} \frac{d}{dz} \left(\frac{X_6}{X_4} \right) - \frac{d^2}{dz^2} \left(\frac{X_6}{X_4} \right) = 0, \end{aligned} \quad (5.5)$$

$$\frac{4}{z} - \frac{4X_6}{zX_4} - 2 \frac{d}{dz} \left(\frac{X_6}{X_4} \right) + m^2 \lambda^s \left(\frac{2}{z} \frac{d\theta}{dz} + \frac{d^2 \theta}{dz^2} \right) = 0. \quad (5.6)$$

It turns out that two coupled equations (5.5) and (5.6) for X_6/X_4 and θ can be solved by choosing two cases:

$$\text{case 1 : } \frac{X_6}{X_4} = 1, \quad \theta = c_2 - \frac{c_1}{z} \quad (5.7)$$

$$\text{case 2 : } \frac{X_6}{X_4} = 1 + \left(2 + c_3 m^2 \lambda^s + \frac{m^2}{H^2} \right) \frac{1}{z^2}, \quad \theta = c_4 - \frac{c_3}{z}, \quad (5.8)$$

where c_i with $i = 1, 2, 3, 4$ are integration constants. Choosing¹ $X_4 = 1$ in case 1 and $X_4 = z^2$ in case 2, the variables X_i with $i = 1, \dots, 6$ in Eq. (5.2) are determined to provide

$$\left[\frac{d^2}{dz^2} + \frac{2}{z} \frac{d}{dz} + 1 + m^2 \left(\frac{1}{z^2 H^2} + \lambda^s \frac{d\theta}{dz} \right) \right] \left[\frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 \right] h_{\mathbf{k}}^s(z) = 0 \quad (5.9)$$

¹We note that in the Einstein-Weyl gravity limit ($\lambda^s \rightarrow 0$), these choices of X_4 give the consistent factorization obtained in the Einstein-Weyl gravity [11]

and

$$\left[\frac{1}{z^2} \frac{d^2}{dz^2} - \frac{2}{z^3} \frac{d}{dz} + \frac{1}{z^2} \right] \left[z^2 \frac{d^2}{dz^2} - 2z \frac{d}{dz} + 2 + z^2 + m^2 \left(\frac{1}{H^2} + \lambda^s z^2 \frac{d\theta}{dz} \right) \right] h_{\mathbf{k}}^s(z) = 0, \quad (5.10)$$

which are Eqs. (3.34) and (3.35), respectively. This implies that the choice of $\theta = c_2 - c_1/z$ could transform the fourth-order equation (5.1) into the product of two second-order equations as (5.9) and (5.10). This factorization unlikely occurs for $\theta = c \ln z$. The same thing happens when one uses η instead of z . This is why we choose $\theta = c/\eta$ for the analysis in the text.

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